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The Jacobi brackets of dissipative field theories

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Outline

Part 1

Introduction

Motivation

Multicontact geometry has been developed to study **non-conservative field theories** in an intrinsic way. On the other hand, the theory of brackets in classical field theory (via **multisymplectic geometry**) is (now!) well developed, and a lot of techniques can been understood in this context: (lower) conservation laws, (higher form) symmetries, the constraint algorihtm... All of this theory heavily employs the **graded nature** of the bracket.

It is then natural to extend these brackets to dissipative fields. In the case of mechanics, the brackets of dissipative systems (when modelled with contact geometry) are naturally related to **conformal symmetries**, namely vector fields satisfying $\mathscr{L}_X\eta = g\eta$.

Motivation

For a **multicontact form** $\Theta \in \Omega^n(M)$, conformal symmetries by vector fields are evident. However, **how about multivectors?** Here we will:

- Define and study **graded brackets** induced by an arbitrary n-form and relate them to the **Poisson brackets** from multisymplectic geometry and the **Jacobi brackets** in contact geometry,
- Develop a **multisymplectization** procedure for the geometry induced by an n-form,
- Introduce an abstract **framework to study multicontact field theories.**

Part 2

Brackets induced by an n-form

Infinitesimal conformal transformations

Let $\Theta \in \Omega^n(M)$ be a smooth n-form on M.

Definition

For $p \leq n$, a multivector field $X \in \mathfrak{X}^p(M)$ is an **infinitesimal conformal transformation** of Θ if there exists a multivector field $V \in \mathfrak{X}^{p-1}(M)$ such that

$$\mathscr{L}_X\Theta = \iota_V\Theta\,,$$

where $\mathscr{L}_X\Theta = \mathrm{d}\iota_X\Theta - (-1)^p\iota_X\,\mathrm{d}\Theta$.

For a vector field $X \in \mathfrak{X}(M)$ the condition reads

$$\mathscr{L}_X\Theta = g\cdot\Theta\,,$$

for $g \in \mathscr{C}^\infty(M)$. We recover the notion of an **infinitesimal conformal symmetry**.

Conformal Hamiltonian forms

Definition

A **conformal Hamiltonian form** is an $(n-p)$-form $\alpha \in \Omega^{n-p}(M)$ such that

$$\iota_{X_\alpha}\Theta = -\alpha\,,$$

for some infinitesimal conformal transformation $X_\alpha \in \mathfrak{X}^p(M)$. We denote by $\Omega^a_H(M,\Theta)$ the space of conformal Hamiltonian a-forms. If the form Θ is clear from the context we will use $\Omega^a_H(M)$.

Note that an $(n-p)$-form α is conformal Hamiltonian if and only if there exist multivector fields $X_\alpha \in \mathfrak{X}^p(M)$ and $V_\alpha \in \mathfrak{X}^{p-1}(M)$ such that

$$\iota_{X_\alpha}\Theta = -\alpha \qquad \text{and} \qquad \iota_{X_\alpha}\mathrm{d}\Theta = (-1)^{p+1}(\mathrm{d}\alpha + \iota_{V_\alpha}\Theta)\,.$$

Theorem

The form $-\iota_{[X_\alpha, X_\beta]}\Theta$ only depends on α and β and is also a conformal Hamiltonian form.

Definition

The **graded Jacobi bracket** (or Jacobi bracket) of two conformal Hamiltonian forms $\alpha \in \Omega^a_H(M)$, $\beta \in \Omega^b_H(M)$ is

$$\{\alpha, \beta\} := -\iota_{[X_\alpha, X_\beta]}\Theta = (-1)^{(p-1)q} \left(\mathscr{L}_{X_\alpha} - \iota_{V_\alpha}\right) \beta \,.$$

Properties of the bracket

Corollary

Let $\alpha \in \Omega^{n-p}(M)$, $\beta \in \Omega^{n-q}(M)$ and $\gamma \in \Omega^{n-r}(M)$ be conformal Hamiltonian forms. Then, the bracket of conformal Hamiltonian forms is **graded skew-symmetric**, satisfies the **graded Jacobi identity**, and the **weak Leibniz rule**, namely

(i) $\{\alpha, \beta\} = -(-1)^{(p-1)(q-1)}\{\beta, \alpha\}$,

(ii) $(-1)^{(p-1)(r-1)}\{\alpha, \{\beta, \gamma\}\} + (-1)^{(r-1)(q-1)}\{\gamma, \{\alpha, \beta\}\} + (-1)^{(q-1)(p-1)}\{\beta, \{\gamma, \alpha\}\} = 0$,

(iii) $\operatorname{supp}\{\alpha, \beta\} \subseteq \operatorname{supp} \alpha \cap \operatorname{supp} \beta$.

Cup product of conformal Hamiltonian forms

It turns out that infinitesimal conformal transformations are **closed under exterior products.** This motivates the following definition.

Definition

The **cup product** of conformal Hamiltonian forms is defined as

$$\alpha \vee \beta := -\iota_{X_\alpha \wedge X_\beta} \Theta \,.$$

Corollary (Graded Leibniz rule)

Given $\alpha \in \Omega_H^{n-p}(M)$, $\beta \in \Omega_H^{n-q}(M)$ and $\gamma \in \Omega_H^{n-r}(M)$, then

$$\{\alpha, \beta \vee \gamma\} = \{\alpha, \beta\} \vee \gamma + (-1)^{(r-1)q} \beta \vee \{\alpha, \gamma\} \,.$$

Some comments

Observe that this endows the set of conformal Hamiltonian forms with a structure of a **Gerstenhaber algebra**, and the map

$$X \mapsto -\iota_X \Theta$$

is a Gerstenhaber algebra homomorphism between infinitesimal conformal transformations and conformal Hamiltonian forms.

Remark. *It is worth noting that, in the case where $\Theta = \eta$ defines a contact form on M, the cup product $f \vee g$ of two functions vanishes. Thus, we only have the weak Leibniz rule in the contact case.*

Part 3

Multicontact structures and multisymplectization

The contact–symplectic equivalence

Definition 1

A **multisymplectic form** is a form $\Omega \in \Omega^{n+1}(M)$ which is closed and non-degenerate.

There is a well-known category equivalence:

$$\{\text{Contact manifolds}\} \cong \{\text{Homogeneous symplectic manifolds}\}\,.$$

In this part we look for a similar equivalence in order to motivate what we will call **multicontact structures**, namely we look for an equivalence

$$\{\text{Multicontact manifolds}\} \cong \{\text{Homogeneous multisymplectic manifolds}\}\,.$$

Homogeneous (pre-)multisymplectic manifolds

Definition

An **homogeneous (pre-)multisymplectic** manifold is a pair $(\widetilde{M}, \Upsilon)$, where $\Upsilon \in \Omega^n(\widetilde{M})$ is such that

- $\Omega = -\,\mathrm{d}\Upsilon$ is a (pre-)multisymplectic form on M, and
- $\exists \Delta \in \mathfrak{X}(\widetilde{M})$, called **a Liouville vector field** satisfying $\iota_\Delta \Omega = -\Upsilon$.

Remark. *A homogeneous symplectic structure is just an exact symplectic structure $\omega = -\,\mathrm{d}\upsilon$, since there always exists Δ such that $\iota_\Delta \omega = -\upsilon$.*

However, this does not hold in multisymplectic geometry since we cannot guarantee the existence of a vector field Δ satisfying $\iota_\Delta \Omega = -\Upsilon$.

(Pre-)multisymplectization

Definition

The **canonical (pre-)multisymplectization** of (M, Θ) is the homogeneous multisymplectic manifold $\widetilde{M} := M \times \mathbb{R}_\times$, with $\Omega = -\mathrm{d}(z\,\Theta)$. Here, $\Delta = z\dfrac{\partial}{\partial z}$ and $\phi = z$.

Theorem

For every infinitesimal conformal transformation $X \in \mathfrak{X}^p(M)$ of Θ, there exists a unique-up-to-$\ker_p \Omega$ multivector field $\widetilde{X} \in \mathfrak{X}^p(\widetilde{M})$ such that

$$\mathscr{L}_{\widetilde{X}}(z\Theta) = 0 \qquad \text{and} \qquad \tau_* \widetilde{X} = X\,.$$

The graded Poisson bracket

On a pre-multisymplectic manifold, there is a **graded Poisson bracket**, defined on the space of **pre-multisymplectic Hamiltonian forms**:

$$\Omega^a_H(M,\Omega) := \{\alpha \in \Omega^a_H(M) \colon \ \mathrm{d}\alpha = \iota_{X_\alpha}\Omega\}\,.$$

Then, there is a well defined bracket:

$$\Omega^a_H(M,\Omega) \otimes \Omega^b_H(M,\Omega) \to \Omega^{a+b-(n-1)}_H(M,\Omega)\,,$$

$\{\alpha,\beta\}_P = \iota_{X_\alpha \wedge X_\beta}\Omega$.

Lifting the Jacobi bracket to the Poisson bracket

Theorem

Let Θ be a nowhere vanishing n-form on M and let $\tau\colon \widetilde{M} \to M$, with $(\widetilde{M}, -\mathrm{d}(z\Theta))$, be the canonical (pre-)multisymplectization of (M, Θ) Then, the map

$$\begin{array}{rccc} \Psi\colon & \Omega^{n-p}_H(M,\Theta) & \longrightarrow & \Omega^{n-p}_H(\widetilde{M}, \Omega = -\mathrm{d}\Upsilon) \\ & \alpha & \longmapsto & (-1)^{p+1} z \cdot \tau^*\alpha \end{array}$$

is well defined and, for every $\alpha \in \Omega^{n-p}_H(M,\Theta)$ and $\beta \in \Omega^{n-q}_H(M,\Theta)$ it satisfies

$$\{\Psi(\alpha), \Psi(\beta)\}_P = \Psi(\{\alpha,\beta\}) + \underbrace{(-1)^q \,\mathrm{d}\Psi(\alpha \vee \beta)}_{\text{exact term}} .$$

Multicontact manifolds

Proposition

If $\ker_1 \mathrm{d}\Theta \neq \{0\}$, then $(M \times \mathbb{R}_\times, \Omega := -\mathrm{d}(z\,\Theta))$ is a non-degenerate homogeneous multisymplectic manifold if and only if

$$\ker_1 \Theta \cap \ker_1 \mathrm{d}\Theta = \{0\}\,.$$

Definition

A **multicontact manifold** is a pair (M, Θ) such that

$$\ker_1 \Theta \cap \ker_1 \mathrm{d}\Theta = \{0\} \qquad \text{and} \qquad \ker_1 \mathrm{d}\Theta \neq \{0\}\,.$$

Condition $\ker_1 \mathrm{d}\Theta \neq \{0\}$ ensures that the canonical pre-multisymplectization of a multicontact manifold (M, Θ) is non-degenerate.

Some comments

Remark. *Let $\eta \in \Omega^1(M)$ such that*

$$\ker_1 \eta \cap \ker_1 \mathrm{d}\eta = \{0\} \qquad \textit{and} \qquad \ker_1 \mathrm{d}\eta \neq \{0\}\,.$$

M is odd-dimensional since $\dim \ker_1 \eta = \dim M - 1$ and $\operatorname{codim} \ker_1 \mathrm{d}\eta$ is even dimensional. The conditions above guarantee that

$$\begin{array}{rccc} \flat\colon & \mathrm{T}\,M & \longrightarrow & \mathrm{T}^*\,M \\ & v & \longmapsto & \iota_v \mathrm{d}\eta + \eta(v)\cdot\eta \end{array}$$

*defines a vector bundle isomorphism, and thus η defines a **contact form**.*

Remark. *Note that a multicontact n-form Θ is necessarily nonvanishing: if $\Theta|_p = 0$, then $\ker_1 \Theta|_p = \mathrm{T}_p\,M$ which would yield $\{0\} = \ker_1 \Theta|_p \cap \ker_1 \mathrm{d}\Theta|_p = \ker_1 \mathrm{d}\Theta|_p \neq \{0\}$, contradiction.*

Part 4

Multicontact field equations

Hamilton–de Donder–Weyl equations

Under certain **suitable conditions**, there exists a multivector field $\mathcal{R} \in \mathfrak{X}^n(M)$, the **Reeb multivector field.** Now, given a hamiltonian $h \in \Omega^n(M)$, we can define the **Hamilton–de Donder–Weyl** equations for a map $\psi\colon X \to M$, where X is and n-dimensional manifold, as

$$\psi^*(\Theta + h) = 0 \qquad \text{and} \qquad \psi^*\iota_\xi(\mathrm{d} + \sigma_h\wedge)(\Theta + h) = 0\,, \quad \forall \xi \in \mathfrak{X}(M)\,.$$

Here, $\sigma_h = \iota_{\mathcal{R}}\,\mathrm{d}h$ is the **dissipation 1-form.**

Remark. *These equations generalize the equations found in the literature of multicontact systems. Indeed, denoting by $\Theta_h = \Theta + h$ the equations now read as*

$$\psi^*\Theta = 0 \quad \textit{and} \quad \psi^*\iota_\xi\overline{\mathrm{d}}\Theta_h = 0\,, \quad \forall \xi \in \mathfrak{X}(M)$$

where $\overline{\mathrm{d}} = \mathrm{d} + \sigma_h\wedge$.

Later, we will see a local computation in the case of fiber bundles $\pi\colon Y \to X$.

Multisymplectization of the dynamics

Let (M, η) be contact and $H \in \mathscr{C}^\infty(M)$. We define a **homogeneous Hamiltonian** on its canonical symplectization $(\widetilde{M} := M \times \mathbb{R}_\times, -\mathrm{d}(z\,\eta))$ as $\widetilde{H}(p, z) := zH(p)$. The **symplectic** Hamiltonian vector field $X_{\widetilde{H}} \in \mathfrak{X}(\widetilde{M})$ is **τ-related** with the **contact** Hamiltonian vector field $X_H \in \mathfrak{X}(M)$.

Definition

Let $(\widetilde{M}, \Omega)$ be a multisymplectic manifold and $\widetilde{h} \in \Omega^n(\widetilde{M})$ be an arbitrary form (which we refer to as the Hamiltonian). Then, a map $\widetilde{\psi} \colon X \to \widetilde{M}$ from an n-dimensional manifold X to $\widetilde{M}$, is said to satisfy the **multisymplectic Hamilton–De Donder–Weyl equations** if

$$\widetilde{\psi}^* \iota_\xi \left(\Omega - \widetilde{h}\right) = 0\,, \quad \forall \xi \in \mathfrak{X}(\widetilde{M})\,.$$

Multisymplectization of the dynamics

Theorem

Let (M, Θ) be variational multicontact and let $(\widetilde{M}, \Omega)$ denote its canonical multisymplectization. Given a good Hamiltonian $h \in \Omega^n(M)$, define the associated homogeneous Hamiltonian as $\widetilde{h}(p, z) := z\tau^* h(p)$. Under technical conditions:

(i) Solutions to the homogeneous HdDW equations project onto solutions to the multicontact HdDW equations.

(ii) Solutions to the multicontact HdDW equations can lifted to solutions to the homogenous HdDW equations provided that the dissipation form is exact over the solutions.

(iii) There is a correspondence between the evolution of observables: for $\alpha \in \Omega^a_H(M)$ conformal Hamiltonian, defining $\widetilde{\alpha}(p, z) := z\,\tau^*\alpha(p)$, we have

$$\widetilde{\psi}^*(\mathrm{d}\widetilde{\alpha}) = (\widetilde{\psi}^* z) \cdot \widetilde{\psi}^* \left(\sigma_h \wedge \alpha + \mathrm{d}\alpha\right) .$$

Part 5

Dissipative field theories

Dissipative field theories

Let's apply all these constructions to the case of **dissipative field theories.**

Let $\pi\colon\ Y \to X$ be a fiber bundle. The **extended phase space** of action dependent field theories is defined as

$$M := \bigwedge_2^n Y \oplus \bigwedge^{n-1} X\,,$$

where $\bigwedge_2^n Y$ denotes the bundle of 2-horizontal n-forms on Y and $\bigwedge^{n-1} X$ denotes the bundle of $(n-1)$-forms on X.

We can find a **canonical multicontact form** on M given by

$$\Theta = \mathrm{d}\Theta_X^{n-1} - \Theta_Y^n\,,$$

where Θ_Y^n and Θ_X^{n-1} denote the canonical Liouville forms on $\bigwedge_2^n Y$ and $\bigwedge^{n-1} X$.

Local expressions

Let (x^μ, y^i) denote fibered coordinates on $\pi\colon Y \to X$, and let $(x^\mu, y^i, p, p_i^\mu, s^\mu)$ denote the coordinates on M. **Any element in** $\bigwedge_2^n Y$ (resp. in $\bigwedge^{n-1} X$) reads

$$p\,\mathrm{d}^n x + p_i^\mu\,\mathrm{d}y^i \wedge \mathrm{d}^{n-1}x_\mu \qquad (\,\text{resp. } s^\mu\,\mathrm{d}^{n-1}x_\mu\,)\,,$$

where $\mathrm{d}^n x = \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^n$ and $\mathrm{d}^{n-1}x_\mu = i_{\frac{\partial}{\partial x^\mu}}\,\mathrm{d}^n x$.
The **canonical multicontact form and its exterior differential** read

$$\Theta = \mathrm{d}s^\mu \wedge \mathrm{d}^{n-1}x_\mu - p\,\mathrm{d}^n x - p_i^\mu\,\mathrm{d}y^i \wedge \mathrm{d}^{n-1}x_\mu\,, \qquad \mathrm{d}\Theta = -\,\mathrm{d}p \wedge \mathrm{d}^n x - \mathrm{d}p_i^\mu \wedge dy^i \wedge \mathrm{d}^{n-1}x_\mu\,.$$

And finally, their **kernels** are

$$\ker_1 \Theta = \left\langle \frac{\partial}{\partial y^i} - p_i^\mu \frac{\partial}{\partial s^\mu}\,, \frac{\partial}{\partial p_i^\mu} \right\rangle \qquad \text{and} \qquad \ker_1 \mathrm{d}\Theta = \left\langle \frac{\partial}{\partial s^\mu} \right\rangle\,,$$

which implies that Θ defines a **multicontact structure.**

Vertical infinitesimal conformal transformations

As discussed before, the best behaved conformal Hamiltonian forms are the ones whose **infinitesimal conformal transformation** lies in $(\operatorname{Im} \flat_\Theta)^{\circ,1}$.

Proposition

For $n > 1$, **vertical infinitesimal conformal transformations** read

$$X = \left(F s^\mu + G^\mu - p_j^\mu \frac{\partial G^\nu}{\partial p_j^\nu} \right) \frac{\partial}{\partial s^\mu} - \frac{\partial G^\mu}{\partial p_i^\mu} \frac{\partial}{\partial y^i} + \left(\frac{\partial F}{\partial y^i} s^\mu + \frac{\partial G^\mu}{\partial y^i} + F p_i^\mu \right) \frac{\partial}{\partial p_i^\mu} + \left(\frac{\partial F}{\partial x^\mu} s^\mu + \frac{\partial G^\mu}{\partial x^\mu} + F p \right) \frac{\partial}{\partial p} ,$$

where F is independent of s^μ, p and p_i^μ, and G^μ satisfies $\dfrac{\partial G^\mu}{\partial p_i^\nu} = \delta_\nu^\mu \dfrac{\partial G^\alpha}{\partial p_i^\alpha}$.

Vertical infinitesimal conformal transformations

Corollary

A Hamiltonian form $\alpha \in \Omega^{n-1}_H(M)$ with vertical infinitesimal conformal transformation has the following local expression:

$$\alpha = (-Fs^\mu - G^\mu)\,\mathrm{d}^{n-1}x_\mu\,,$$

where $F = F(x, y)$, and G^μ satisfies $\dfrac{\partial G^\mu}{\partial p^\nu_i} = \delta^\mu_\nu \dfrac{\partial G^\alpha}{\partial p^\alpha_i}$.

Elementary examples of conformal Hamiltonian forms

Form	Infinitesimal conformal transformation	Conformal factor
$s^\mu \,\mathrm{d}^{n-1} x_\mu$	$-s^\mu \dfrac{\partial}{\partial s^\mu} - p_i^\mu \dfrac{\partial}{\partial p_i^\mu} - p \dfrac{\partial}{\partial p}$	-1
$y^i \,\mathrm{d}^{n-1} x_\mu$	$-y^i \dfrac{\partial}{\partial s^\mu} - \dfrac{\partial}{\partial p_i^\mu}$	0
$p_i^\mu \,\mathrm{d}^{n-1} x_\mu$	$\dfrac{\partial}{\partial y^i}$	0
$\mathrm{d}^{n-1} x_\mu$	$-\dfrac{\partial}{\partial s^\mu}$	0

Jacobi brackets of the elementary examples

$\{\cdot,\cdot\}$	$s^\nu \,\mathrm{d}^{n-1}x_\nu$	$y^j \,\mathrm{d}^{n-1}x_\nu$	$p^\nu_j \,\mathrm{d}^{n-1}x_\nu$	$\mathrm{d}^{n-1}x_\nu$
$s^\mu \,\mathrm{d}^{n-1}x_\mu$	0	$y^j \,\mathrm{d}^{n-1}x_\nu$	0	$\mathrm{d}^{n-1}x_\mu$
$y^i \,\mathrm{d}^{n-1}x_\mu$	$-y^i \,\mathrm{d}^{n-1}x_\mu$	0	$\delta^i_j \,\mathrm{d}^{n-1}x_\nu$	0
$p^\mu_i \,\mathrm{d}^{n-1}x_\mu$	0	$-\delta^j_i \,\mathrm{d}^{n-1}x_\mu$	0	0
$\mathrm{d}^{n-1}x_\mu$	$-\,\mathrm{d}^{n-1}x_\mu$	0	0	0

Hamiltonians

Now, our goal is to write the evolution of the previous Hamiltonian forms with a fixed Hamiltonian (and hence fixed dynamics) using the studied geometry.

Intrinsically, a Hamiltonian is given by a **Hamiltonian section**

$$h\colon \bigwedge_2^n Y \big/ \bigwedge_1^n Y \oplus \bigwedge^{n-1} X \longrightarrow M = \bigwedge_2^n Y \oplus \bigwedge^{n-1} X\,,$$

which then induces a multicontact structure on $\bigwedge_2^n Y / \bigwedge_1^n Y \oplus \bigwedge^{n-1} X$ using $\Theta_h = h^*\Theta$.

As stated before, we prefer to work with the canonical multicontact structure on M and think the Hamiltonian as $\widetilde{h} = \tau^*\Theta_h - \Theta$, where $\tau\colon M \to \bigwedge_2^n Y / \bigwedge_1^n Y \oplus \bigwedge^{n-1} X$ denotes the projection, hence fixing the geometry.

Local expressions

The canonical coordinates on $\bigwedge_2^n Y / \bigwedge_1^n Y \oplus \bigwedge^{n-1} X$ are $(x^\mu, y^i, p_i^\mu, s^\mu)$ and, hence, the local expression of the Hamiltonian section will be $p = -H(x^\mu, y^i, p_i^\mu, s^\mu)$, so that

$$\Theta_h = \mathrm{d}s^\mu \wedge \mathrm{d}^{n-1}x_\mu + H\,\mathrm{d}^n x - p_i^\mu\,\mathrm{d}y^i \wedge \mathrm{d}^{n-1}x_\mu\,,$$

and

$$\widetilde{h} = (p + H)\,\mathrm{d}^n x\,.$$

This implies that $\widetilde{h}$ is a good Hamiltonian, so that it could define the following dynamics for some $\psi \colon X \to M$:

$$\psi^*(\Theta + h) = 0 \qquad \text{and} \qquad \psi^* \iota_\xi(\mathrm{d} + \sigma_h \wedge)(\Theta + h) = 0\,, \quad \forall \xi \in \mathfrak{X}(M)\,,$$

where $\sigma_h = \dfrac{\partial H}{\partial s^\mu}\,\mathrm{d}x^\mu$ is the associated dissipation 1-form.

Multicontact Hamilton–de Donder–Weyl equations

The **local expression of the multicontact HdDW equations** is

$$
\begin{aligned}
\psi^*(\mathrm{d}s^\mu \wedge \mathrm{d}^{n-1}x_\mu) &= \psi^*\left(\left(p_i^\mu \frac{\partial H}{\partial p_i^\mu} - H\right)\mathrm{d}^n x\right)\,,\\
\psi^*(\mathrm{d}y^i \wedge \mathrm{d}^{n-1}x_\mu) &= \psi^*\left(\frac{\partial H}{\partial p_i^\mu}\,\mathrm{d}^n x\right)\,,\\
\psi^*(\mathrm{d}p_i^\mu \wedge \mathrm{d}^{n-1}x_\mu) &= \psi^*\left(-\left(\frac{\partial H}{\partial y^i} + \frac{\partial H}{\partial s^\mu}p_i^\mu\right)\mathrm{d}^n x\right)\,.
\end{aligned}
$$

In particular, if we look for **sections** $\psi\colon X \to M$, we obtain the so-called multicontact Hamilton–de Donder–Weyl equations for dissipated field theories:

$$
\frac{\partial s^\mu}{\partial x^\mu} = p_i^\mu \frac{\partial H}{\partial p_i^\mu} - H\,, \qquad \frac{\partial y^i}{\partial x^\mu} = \frac{\partial H}{\partial p_i^\mu}\,, \qquad \frac{\partial p_i^\mu}{\partial x^\mu} = -\left(\frac{\partial H}{\partial y^i} + \frac{\partial H}{\partial s^\mu}p_i^\mu\right)\,.
$$

Part 6

Conclusions

Conclusions

In this talk, we have:

- Defined and study **brackets** induced by an arbitrary n-form and related them to the **Poisson brackets** from multisymplectic geometry and the **Jacobi brackets** in contact geometry.
- Developed a **multisymplectization** procedure for the geometry induced by an n-form.
- Introduced an abstract **framework to study multicontact field theories**.

For further work, we aim to understand the relationship of the brackets to dissipated quantitities and the dynamics.

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Muchas gracias!

Questions?

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